

2m/MTH-150 Syllabus-2023

2026

(May-June)

FYUP : 2nd Semester Examination

MATHEMATICS

(Minor)

(**Fundamental Mathematics—II**)

(MTH-150)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer **four** questions, taking **one** from each Unit.

UNIT—I

1. (a) If by transformation from one set of rectangular axes to another with the same origin, the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$. 4

- (b) Prove that the equation

$$2x^2 + xy - 6y^2 - 6x + 23y - 20 = 0$$

represents a pair of straight lines. Find the angle between them. 5

(c) Reduce

$$144x^2 - 120xy + 25y^2 + 243x + 448y - 113 = 0$$

to the standard form.

6

(d) What type of conic do the following equations represent? $1\frac{1}{2} + 1\frac{1}{2} = 3$

(i) $17x^2 + 12xy + 8y^2 - 46x - 28y + 17 = 0$

(ii) $2x^2 - 5xy + 2y^2 + 3x - 4y + 1 = 0$

2. (a) The equation

$$3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$$

is transformed to $4x^2 + 2y^2 + 1 = 0$ when referred to rectangular axes through the point (2, 3). Find the inclination of the latter axes to the former.

5

(b) Prove that the equation of the tangent to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy = 0$$

at the origin is $gx + fy = 0$.

3

(c) If the straight lines represented by the equation

$$x^2(\tan^2 \phi + \cos^2 \phi) - 2xy \tan \phi + y^2 \sin^2 \phi = 0$$

make angles α and β with the positive direction of the x -axis, show that $\tan \alpha - \tan \beta = 2$.

6

(d) Find the diameter of the conic $15x^2 - 20xy + 16y^2 = 1$ conjugate to the diameter $y + 2x = 0$.

4

UNIT—II

3. (a) Find the equation of the cone reciprocal to the cone

$$19x^2 + 11y^2 + 3z^2 + 6yz - 10zx - 26xy = 0 \quad 4$$

(b) Prove that the planes $x + 2y + 2z = 0$, $2x + y - 2z = 0$ are at right angles. 3(c) Show that the spheres $x^2 + y^2 + z^2 = 25$ and

$$x^2 + y^2 + z^2 - 24x - 40y - 18z + 225 = 0$$

touch externally and find the point of contact.

6

(d) Find the centre and radius of the circle given by the equations $x + 2y + 2z = 15$ and $x^2 + y^2 + z^2 - 2y - 4z = 11$.

6

4. (a) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$$

$$x - 2y + 4z - 9 = 0$$

and the centre of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0 \quad 5$$

(b) Find the equation of the plane passing through the point (2, -3, 4) and parallel to the plane $2x - 5y - 7z + 15 = 0$.

4

- (c) Find the coordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4$ at which the tangent plane is parallel to the plane $2x - y + 2z = 1$. 5
- (d) Find the equation of the cone, whose vertex is the origin and the guiding curve is given by $x + 2y + 3z = 4$ and $5x^2 + 7y^2 - 3z + 2 = 0$. 5

UNIT—III

5. (a) Discuss the existence of the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$, where

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}; & \text{when } (x, y) \neq (0, 0) \\ 0; & \text{when } (x, y) = (0, 0) \end{cases} \quad 5$$

- (b) Prove that the function

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x - y}; & \text{when } x \neq y \\ 0; & \text{when } x = y \end{cases}$$

is not continuous at $(0, 0)$, but both partial derivatives exist at $(0, 0)$. 6

- (c) Show that

$$\frac{\partial^2 u}{\partial x \partial y} \text{ and } \frac{\partial^2 u}{\partial y \partial x}$$

are identical when $u = x \log y$ ($y > 0$). 4

- (d) Find the domain and range of

$$g(x, y) = \frac{xy}{x^2 + y^2} \quad 4$$

6. (a) State Euler's theorem for homogeneous functions of two variables. Hence verify

$$\text{Euler's theorem for } u = \frac{x+y}{x-y}. \quad 1+4=5$$

- (b) Are the following functions homogeneous? If so, find their degree of homogeneity : $1+1=2$

$$(i) f(x, y) = x^2 + x^2 \log\left(\frac{y}{x}\right)$$

$$(ii) f(x, y) = \frac{x^2}{y} + \frac{y^2}{x}$$

- (c) If $u = \tan^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u. \quad 5$$

- (d) Find the first-order partial derivatives

$$\text{of } f(x, y) = \frac{x+y-1}{x+y+1} \text{ at } (2, 1). \quad 3$$

- (e) Find f_x , f_y , f_{xy} and f_{xx} for the function $\log(x+y)$. 4

UNIT—IV

7. (a) If $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors, then show that

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$$

and hence show that $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are non-coplanar.

5

- (b) Find the directional derivative of the function $f = xy + yz + zx$ in the direction of the vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ at the point (3, 1, 2).

5

- (c) Find the equation of the tangent plane to the surface $yz - zx + xy + 5 = 0$ at the point (1, -1, 2).

5

- (d) (i) Prove that the vector

$$\vec{f} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$$

is solenoidal.

- (ii) Determine the constant h so that the vector

$$\vec{f} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + hz)\hat{k}$$

is solenoidal.

2+2=4

8. (a) If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, show that

$$(i) \vec{r} \times \frac{d\vec{r}}{dt} = \omega \vec{a} \times \vec{b}$$

$$(ii) \frac{d^2 \vec{r}}{dt^2} = -\omega^2 \vec{r}$$

where $\vec{a}$ and $\vec{b}$ are constant vectors.

2+3=5

- (b) Show that $\text{grad } f(r) \times \vec{r} = \vec{0}$, where

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}.$$

5

- (c) If

$$\vec{A} = t^3 \hat{i} + 2t \hat{j} + e^t \hat{k}$$

$$\text{and } \vec{B} = \sin t \hat{i} - \cos t \hat{j} + t \hat{k}$$

find—

$$(i) \frac{d}{dt}(\vec{A} \cdot \vec{B})$$

$$(ii) \frac{d}{dt}(\vec{A} \times \vec{B})$$

2+3=5

- (d) Find $\text{div } A$ and $\text{curl } A$, if

$$A(x, y, z) = xy^2z^3\hat{i} + (y^2 + z^2)\hat{j} + x^2z^2\hat{k}$$

2+2=4
